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Lagrangian Random Choice Method for Steady Two-Dimensional Supersonic/Hypersonic Flow

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Abstract

GLIMM'S random choice method has been successfully applied to numerically solve the two-dimentional steady Euler equations of gas dynamics based on a new Lagrangian formulation. The method is easy to program, fast to execute, yet it is very accurate and robust. It requires no grid generation, resolves slipline and shock discontinuities crisply, can handle boundary conditions most easily, and is applicable to hypersonic as well as supersonic flow.

Contents

The random choice method (RCM) of Glimm[1] for constructive proof of existence of solutions to nonlinear hyperbolic systems of conservation laws has been developed by Chorin[2] as a practical computational method for finding such solutions, especially to the one-dimensional Euler equations of gas dynamics. Further improvements and applications of the method have been reported in Ref. 3. In essence, the solution is advanced in time by a sequence of operations that includes the solution of Riemann problems and a stochastic sampling procedure. However, the very desirable properties of RCM—no diffusive errors from spatial averaging—do not seem to persist for flows in two space dimensions.[3] For two-dimensional steady supersonic flow, one of the space variables can be treated as a time-like one and RCM can be applied in a way similar to that for one-dimensional unsteady flow.

Recently, the present authors[4,5] have developed a computational method for solving the steady Euler equations based on the new Lagrangian formulation of Hui and Van Roessel.[6] This is made possible by the introduction of the Lagrangian time τ, which effectively reduces a two-dimensional steady supersonic flow problem to that of a one-dimensional unsteady flow. Applications of the Godunov/TVD scheme[4,5] have shown that the new Lagrangian method is superior to the Eulerian method.

The purpose of this synoptic is to apply Glimm's random choice method to compute two-dimensional steady inviscid supersonic/hypersonic flows based on the new Lagrangian formulation.

New Lagrangian Conservation Form

For two-dimensional steady flow, using the Lagrangian time τ and the stream funciton ξ as independent variables, the Euler equations are written in conservation form[4]

$$\frac{\partial \boldsymbol{E}}{\partial \tau} + \frac{\partial \boldsymbol{F}}{\partial \xi} = 0 \qquad (1)$$

where

$$E = (K, H, Ku + pV, Kv - pU, U, V)^T$$

$$F = (0, 0, -pv, -pu, -u, -v)^T$$

$$U = \frac{\partial x}{\partial \xi} \qquad V = \frac{\partial y}{\partial \xi}$$

$$K = \rho\,(uV - vU), \qquad H = \frac{1}{2}(u^2 + v^2) + \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$$

and p, ρ, u, and v are the pressure, density, and the x- and y-component of velocity, respectively. The first four equations of (1) express conservation of mass, energy, and momentum, respectively; whereas the last two equations are the compatibility conditions that are a unique feature of the Lagrangian formulation, representing the geometrical deformation of the fluid particles. We restrict our attention to flows that are supersonic everywhere, so that Eq. (1) can be solved by a numerical scheme that marches in τ.

Application of Glimm's Method

We now sketch the application of Glimm's RCM in the new Lagrangian formulation. To solve the initial-boundary value problem of Eq. (1) the computational domain is divided by streamlines, $0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \xi_N$, into N cells (Fig. 1). The solution is to be evaluated for every cell j, $j = 1, 2, \ldots N$, at time $\tau_1, \tau_2, \tau_3, \ldots$.

For $\tau = \tau_n$ the flow variables $\boldsymbol{Q} = (p, \rho, u, \text{and } v)^T$ are assumed given and constant within each cell j and denoted as $\boldsymbol{Q}_j$. For $j = 1, 2, \ldots, N-1$, a sequence of Riemann problems for neighboring pairs of cells with initial data $\boldsymbol{Q} = \boldsymbol{Q}_{j+1}$, $(\xi > \xi_j)$ and $\boldsymbol{Q} = \boldsymbol{Q}_j$, $(\xi < \xi_j)$ are then solved.

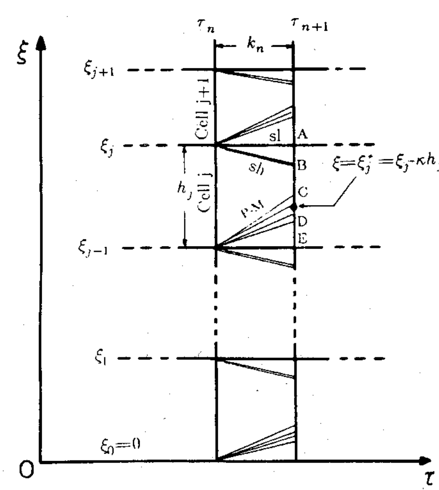

Fig. 1 Computational space and mesh.

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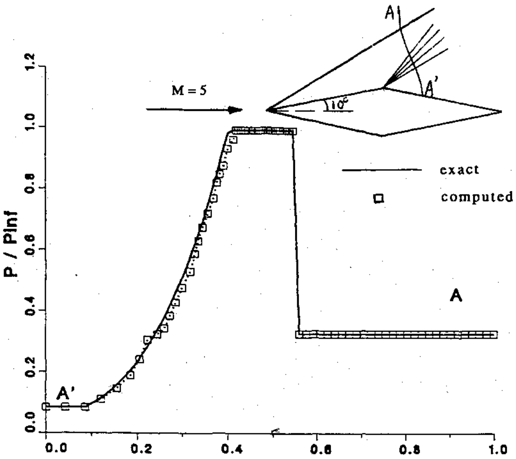

Fig. 2 Diamond shape airofoil with sudden compression corner of 10-deg presure along AA′.

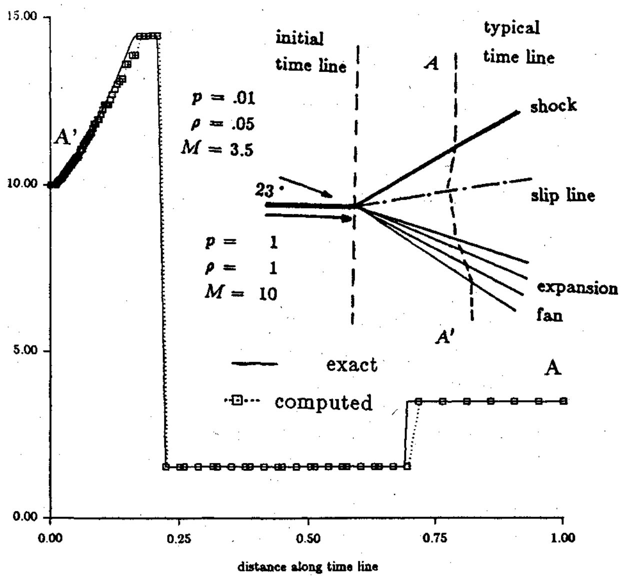

Fig. 3 Computed solution for a Riemann problem, Mach number along a typical timeline AA′.

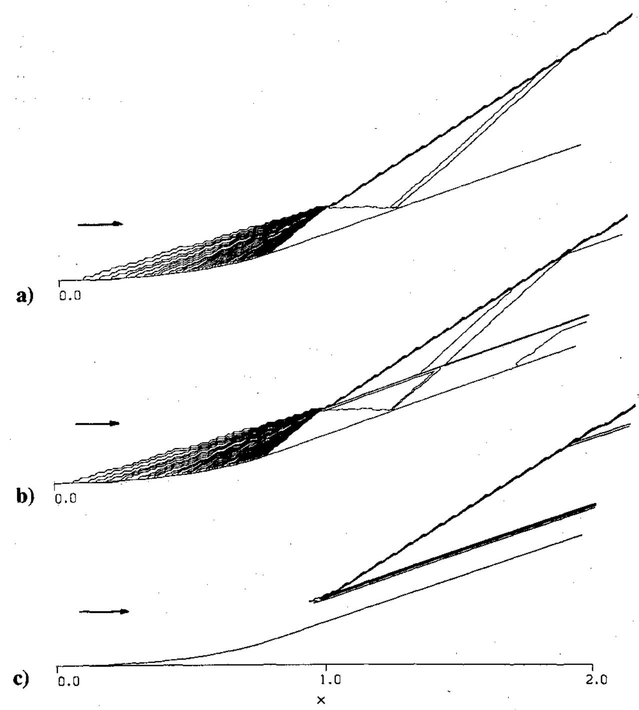

Fig. 4 Sudden formation of shock wave and slipline in the interior of the flowfield as captured by the new lagrangian RCM: a) isobars, b) density contours, and c) entropy contours.

The Riemann solution may consist of a shock (sh), a slip line (sl) and a Prandtl-Meyer expansion (PM) (see Fig. 1). The slip line, being also a streamline, must coincide with the interface between cells j and $j+1$. We choose time step size, $k_n = \tau_{n+1} - \tau_n$, small enough so that the waves resulting from the interactions at the boundaries of the cell do not intersect with each other within the cell (Courant-Friedrichs-Lewy condition). The flow in cell j at time τ_{n+1} is depicted in Fig. 1; it typically consists of a uniform flow in region AB, a shock at B, the original uniform flow Q_j in region BC, the Prandtl-Meyer expansion region CD, and the uniform flow in region DE. In any cell, there are at most five such solution regions. This is computed for every cell j, $j = 1, 2, \ldots N$, at time τ_{n+1}.

Then we use the Van der Corput random number generator[3] to generate a pseudo random number κ, uniformly distributed in (0,1). A simple linear relation maps (0,1) onto the Cell j ($j = 1, 2, \ldots N$) and κ determines which of the above solution regions is sampled as the solution for Cell j. Starting with $\tau_0 = 0$, the procedure described above is repeated to advance the solution to τ_1, τ_2, $\tau_3, \ldots$.

Examples

The robustness and accuracy of the new Lagrangian random choice method are tested in several examples. Examples 1 and 2 are illustrated in Figs. 2 and 3, respectively. Example 3 (Fig. 4) shows a $M = 4$ supersonic flow past a special concave curve ("Γ" curve)[5] with a sudden birth of shock, slipline and expansion fan. More examples can be found in the full paper.

Concluding Remarks

The new Lagrangian random choice method is easy to program, fast to execute, yet it is very accurate and robust. In comparison with Eulerian formulation, it requires no grid generation, produces exact smooth solid boundary shapes, and the sampling procedure is simplified, since one only needs to choose among at most[5] solution regions. In comparison to the deterministic approach of the new Lagrangian formulation[4–5] the random choice method requires no special procedures even at sharp boundary corners.

References

[1]Glimm, J., "Solution in the Large for Nonlinear Hyperbolic Systems of Equations," *Communications of Pure and Applied Mathematics*, Vol. 18, No. 4, 1965, pp. 697–715.

[2]Chorin, A. J., "Random Choice Solution of Hyperbolic Systems," *Journal of Computational Physics*, Vol. 22, No. 4, 1976, pp. 517–536.

[3]Colella, P., "Glimm's Method for Gas Dynamics," *SIAM Journal of Scientific and Statistical Computing*, Vol. 3, No. 1, 1982, pp. 76–110.

[4]Loh, C. Y., and Hui, W. H., "A New Lagrangian Method for Steady Supersonic Flow Computation Part I: Godunov Scheme," *Journal of Computational Physics*, Vol. 89, No. 1, 1990, pp. 207–240.

[5]Hui, W. H., and Loh, C. Y., "A New Lagrangian Method for Steady Supersonic Flow Computation, Part II: Slipline Resolution," *Journal of Computational Physics*, Vol. 103, No. 2, 1992, pp. 450–464.

[6]Hui, W. H., and Van Roessel, H. J., "Unsteady Three Dimensional Flow Theory via Material Functions," NATO AGARD Symposium on Unsteady Aerodynamcis—Fundamentals and Application to Aircraft Dynamics, Gottingen, W. Germany, May, 1985, CP-386, Paper S1.